

## ANALYSIS OF COMBINED CONDUCTION AND RADIATION IN PARTICIPATING MEDIUM WITHIN TWO-DIMENSIONAL RECTANGULAR ENCLOSURES

Kunja Bihari Sahu, Prakash Ghose and Purna Chandra Mishra

KIIT University, Bhubaneswar-751024, Odisha, India

### ABSTRACT

The study of steady state combined conduction and radiation heat transfer with participating media in rectangular enclosures is carried out numerically. The bottom surface is kept at a higher absolute temperature and others at 50% of it.  $P_1$  approximation model is employed for radiative transport equation. The radiative transport and energy conservation equations are discretised with finite elements using Galerkin's method. The aspect ratio is varied from 0.2 to 3 for various radiation-conduction parameters (1, 10 and 100). The effects of radiation conduction parameter, single scattering albedo at various aspect ratios on temperature distribution within the enclosure are discussed.

**Keywords:**  $P_1$  approximation, Finite elements, Galerkin's method, Isotropic scattering.

### 1. INTRODUCTION

The study of combined conduction and radiation heat transfer in a participating medium has attracted considerable interest due to its important applications in areas like multilayer insulation with fillers, fire protection etc. An exact analytical solution to the highly non-linear integro-differential radiative transport equation is almost impossible to find. Further the coupling of energy conservation and radiative transport equations in combined mode of conduction and radiation heat transfer requires an iterative solution procedure. The temperature distribution and heat transfer are influenced by the aspect ratio of the enclosure. In the present study, the combined conduction and radiation heat transfer has been analyzed within rectangular enclosures of various aspect ratios.

Many researchers have developed approximate solutions to radiative transport equation (RTE). In the early studies most of the analyses were made for one-dimensional gray medium problems. Love and Grosh, 1965 studied numerically, the combined heat transfer in one-dimensional emitting, absorbing and non-scattering planar gray media. Yuen and Wang, 1980 investigated the influence of the anisotropic scattering on combined heat transfer in one-dimensional planar geometry. It was found that anisotropic scattering has a significant effect on the total heat flux and the temperature profile in the medium. Benim, 1988 also used the finite element method for solving RTE for

one-dimensional problems applying moment method. Ratzel, 1983 reported the results of a study to two-dimensional radiation in absorbing emitting media using  $P_N$  approximation method for isothermal boundary conditions and concluded that  $P_N$  method was reasonably accurate. Mahapatra et al., 1999 analyzed the problem of combined conduction and radiation in absorbing, emitting gray medium inside a square enclosure.  $P_1$ -approximation has been applied to the radiative transport equation. The radiative transport equation along with the equation of energy conservation has been numerically discretised by finite difference method. The influences of radiation conduction parameter, optical thickness of the medium and surface emissivity on the temperature distributions and heat flux were also discussed. Razzaque et al., 1984 used finite element method for coupled radiative and conductive heat transfer problem inside a rectangular enclosure with isothermal walls with distributed energy sources. Problems involving lower surface emissivity values of the wall and high values of conduction radiation parameter were not been considered in the work. Ho and Ozisik, 1988 analyzed the combined conduction and radiation in an absorbing, emitting and isotropically scattering medium inside a rectangular enclosure with isothermal walls. The Galerkin method was used to obtain the solution to the radiation part of the problem. In the present study, the combined conduction and radiation in rectangular enclosures has been considered.

The bottom wall of the enclosure is at higher temperature and all other walls are at half of the hot wall temperature. The influences of radiation-conduction parameter, aspect ratio and radiative properties on temperature distributions and heat transfer have been discussed.

## 2. ANALYSIS

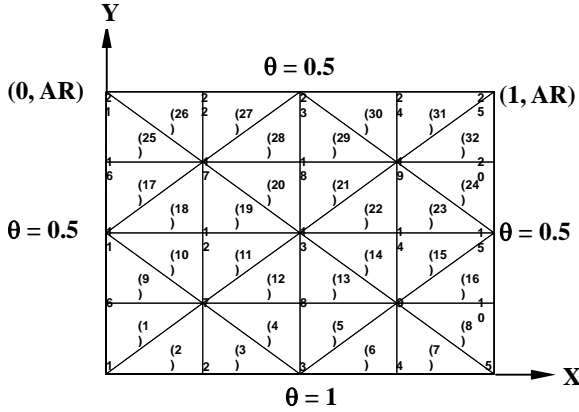


Fig 1. Computational domain and coordinate system.

Consider a rectangular cavity of width  $L$  and height  $H$ . The medium of the enclosure which is an absorbing, emitting and isotropically scattering one is assumed to be of constant property. The bottom wall of the cavity is isothermal at temperature  $T_H$  K and all other walls are at 50% of it. The corresponding non-dimensional temperatures are  $\theta=1$  and  $\theta=0.5$  respectively as depicted in Fig. 1.

For 2-D steady state combined conduction and radiation without internal heat generation, the energy conservation equation can be expressed as,

$$\alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{1}{\rho C_p} \nabla \cdot q_r = 0 \quad (1)$$

The boundary conditions are  
 $T=T_H$  at  $y=0$ ;  $0 < x < L$  and  
 $T=0.5 T_H$  at  $y=H$ ;  $0 < x < L$   
 $T=0.5 T_H$  at  $x=0, L$ ;  $0 \leq y \leq H$ .

The radiative transport equation for an absorbing, emitting and scattering medium under quasi-steady condition expressed by Modest, 1993 is

$$\hat{s} \cdot \nabla I + I = (1 - \omega) I_b + \frac{\omega}{4\pi} \int_{4\pi} I(\hat{s}') \phi(\hat{s} \cdot \hat{s}') d\Omega' \quad (2)$$

Using  $P_1$  approximation method (Modest, 1993), the radiative transport equation is reduced to

$$\nabla^2 \cdot G - (1 - \omega)G = -(1 - \omega)4\pi I_b \quad (3)$$

The boundary conditions are

$$-\frac{(2 - \varepsilon)}{\varepsilon} \cdot \frac{2}{(3 - A_1 \omega)} \hat{n} \cdot \nabla G + G = 4\pi I_{bw} \quad (4)$$

at all walls (Marshak's boundary conditions, Modest, 1993)

The following parameters are used for expressing above governing equations in dimensionless form.

$$X = \frac{x}{L}, Y = \frac{y}{H}, G^* = \frac{G}{4\sigma T_H^4}, \theta = \frac{T}{T_H},$$

$$Q_r = \frac{q_r}{4\sigma T_H^4}, RC = \frac{\sigma T_H^4}{(kT_H/L)}$$

Using dimensionless quantities as above, the energy conservation equation (1) can be expressed as,

$$\nabla^2 \theta - 4(RC) \nabla \cdot \vec{Q}_r = 0 \quad (5)$$

The boundary conditions are

$$\theta=1, \quad \text{at } Y=0 \text{ for } 0 < X < 1 \quad (6a)$$

$$\theta=0.5, \quad \text{at } Y=AR \text{ for } 0 < X < 1 \quad (6b)$$

$$\theta=0.5, \quad \text{at } X=0, 1 \text{ for } 0 \leq Y \leq 1 \quad (6c)$$

The radiative transport equation is expressed as

$$\frac{\partial^2 G^*}{\partial X^2} + \frac{\partial^2 G^*}{\partial Y^2} + C(\theta^4 - G^*) = 0 \quad (7)$$

where,  $C = \tau^2(1 - \omega)$

The boundary conditions are

$$-\frac{(2 - \varepsilon)}{\varepsilon} \cdot \frac{2}{3} \hat{n} \cdot \nabla G^* + \tau G^* - \tau \theta_w^4 = 0 \quad (8)$$

## 3. NUMERICAL PROCEDURE

The governing equations are solved by finite elements. The computational domain is represented by 21 x 21 nodes and 800 linear triangular elements for  $AR=1$ . Keeping the number of divisions constant along the width of the cavity (x-direction), the number of divisions along the height (y-direction) is varied to keep the grid size almost same for different aspect ratios.

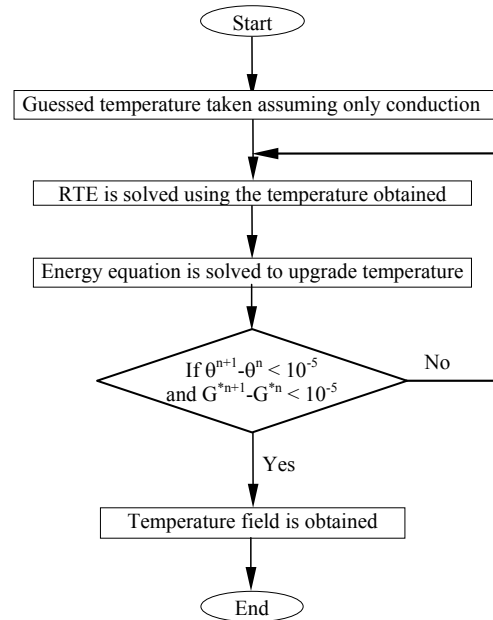


Fig 2. Flow chart for solving the problem

The element equations are developed first using Galerkin's method and then assembled in order to get equations for the whole computational domain. The necessary boundary conditions are imposed, which modifies both the conductivity matrix and heat rate vector appropriately. The governing equations for energy conservation (5) and radiative transport equation (7) are coupled through source terms. The equations for boundary conditions (6) and (8) are also interlocked. Therefore the solution of the above equations can only be obtained through iteration. The flow chart of numerical procedure has been outlined in Fig. 2. During the numerical investigation, the authors experienced that, under-relaxation of momentum equation and energy equation is required for obtaining the convergence.

#### 4. RESULTS AND DISCUSSION

Numerical simulations are performed on absorbing, emitting and scattering media within rectangular enclosures of various aspect ratios to study the effects of various parameters on temperature distribution and heat transfer for combined conduction and radiation phenomenon. The results are discussed as follows:

##### Effect of radiation on center temperature

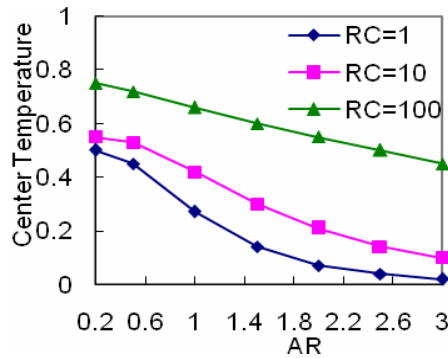


Fig 3. Variation of center temperature with aspect ratio

Heat is supplied by the bottom wall and received by all other walls. Figure 3 shows the variation of center temperature  $T(0.5, 0.5*AR)$  with aspect ratio for  $RC=1, 10$  and  $100$ . The heat transfer from bottom hot wall raises the center temperature whereas that through the vertical walls lowers the center temperature. Keeping the width constant, increase in  $AR$  implies increase in height of the enclosure. The heat loss through the vertical walls is increased with  $AR$ , which results in decrease in center temperature. For higher values of radiation conduction parameter the center temperature increases due to higher radiation effect.

##### Effect of aspect ratio on vertical centerline temperature

For a given  $RC$ , the total radiation heat transfer from the bottom wall is constant, but the total conduction heat loss from the vertical walls depend on the aspect ratio since the height of the vertical walls is changed while the length of the bottom wall is constant. Figure 4 shows the

variation of vertical centerline temperatures with aspect ratio. The heights of the enclosures are non-dimensionalised with the aspect ratio (i.e.,  $Y/AR$ ) to take into account different heights while comparing the vertical centerline temperatures.

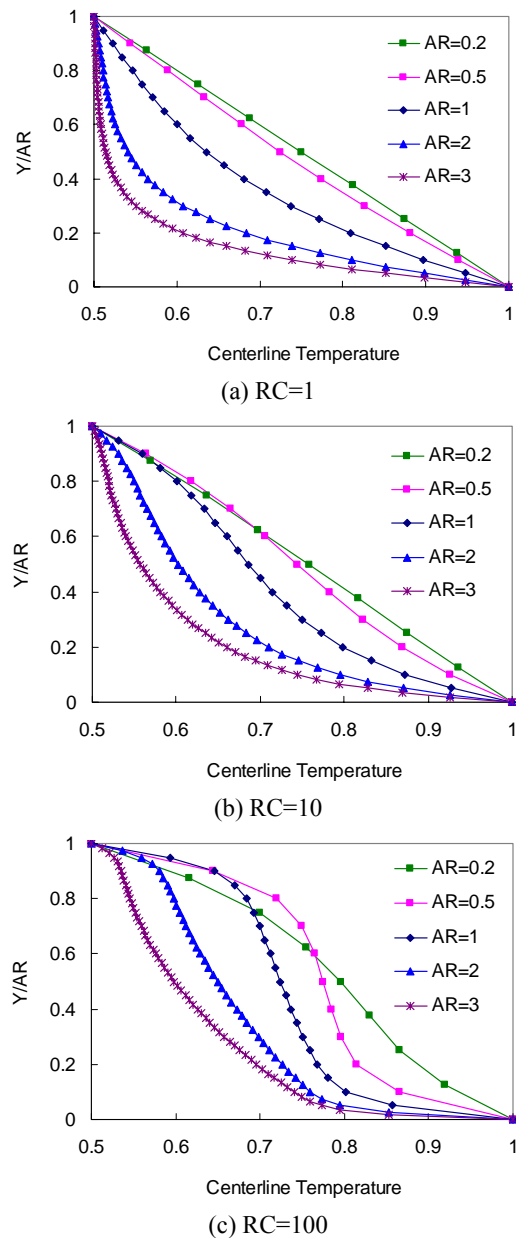


Fig. 4 effect of aspect ratio on vertical centerline temperature for (a)  $RC=1$ , (b)  $RC=10$  AND (c)  $RC=100$

At  $RC=1$  and  $10$ , the plots of centerline temperatures are almost straight for lower values of  $AR$  ( $0.2$  and  $0.5$ ). But the concavity of the plots for higher aspect ratios (say,  $2$  and  $3$ ) at  $RC=1$  is more than that for corresponding aspect ratios at  $RC=10$  as evident from Figs. 4 a and b.

The total radiation from the bottom wall is comparable with the total heat loss from vertical walls at lower  $AR$ . But with increase in the aspect ratio, the total

heat loss from the vertical walls increases thereby decreasing the centerline temperature compared to that in a plane slab. This increases the concavity of the curves. This is also evident from the isotherms for  $AR=0.5$  and  $2$  at  $RC=1$  (Figs. 5 a and b). The isotherms are almost equally spaced from bottom to top wall for the former whereas they are closely spaced near bottom wall and very widely spaced towards top wall for the latter. This is because the energy transfer from the bottom wall towards the top wall is effective for smaller aspect ratios, but it is not so for larger aspect ratios owing to small radiation effect.

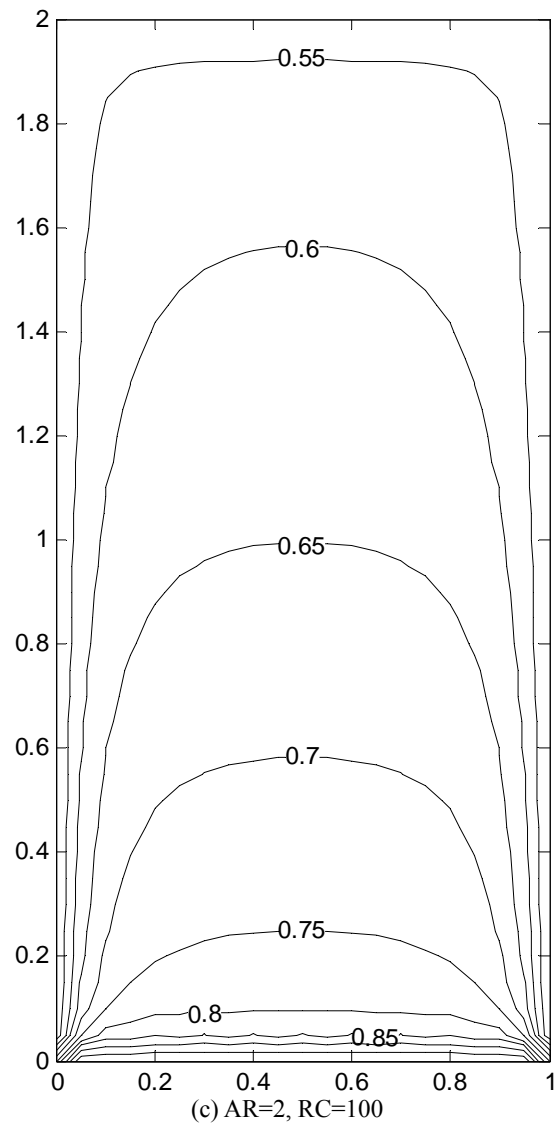
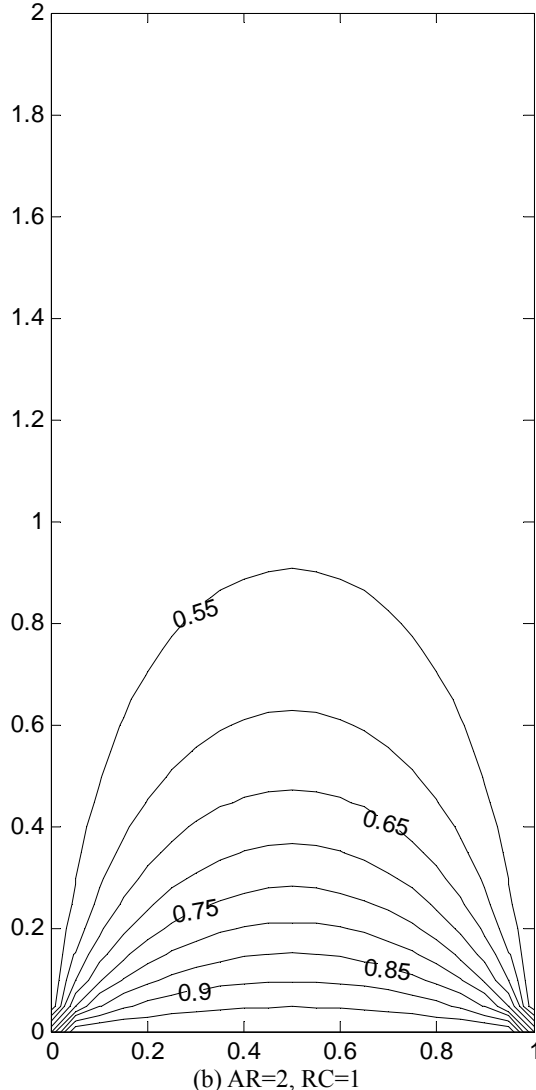
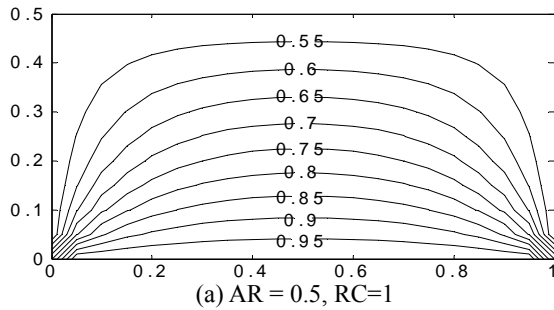


Fig. 5 isotherms for (a)  $AR = 0.5, RC=1$   
(b)  $AR=2, RC=1$  AND (c)  $AR=2, RC=100$

For a given aspect ratio, the radiation heat transfer from bottom wall increases with increase in  $RC$ , which makes the temperature gradient uniform (Fig. 5 b and c).

At  $RC=100$ , the conduction heat transfer is negligible compared to radiation heat transfer. So the temperature distribution is mainly governed by the radiation phenomena. Figure 4 c shows that the vertical centerline temperature near the bottom wall is decreasing continuously with increase in  $AR$ . This is because the portion of radiation heat energy transferred from the bottom wall to the vertical walls increases with  $AR$ . This is corroborated with the higher temperature gradient near the walls as seen from Fig. 5 c. The core is heated by the radiations from the bottom wall making the temperature somewhat uniform. This effect is not more prominent for  $AR=0.2$  as the dimension of the vertical walls is very less compared to that of the horizontal walls.

### Effect of single scattering albedo on vertical centerline temperature distribution

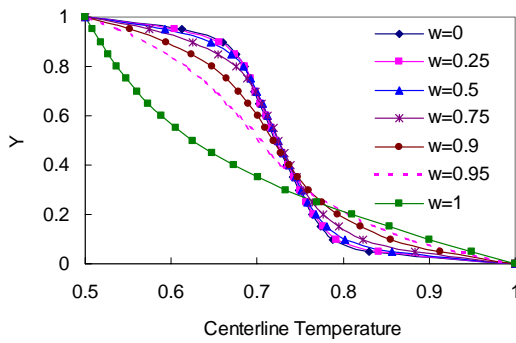


Fig 6. effect of single scattering albedo on vertical centerline temperature (AR=1, RC=100)

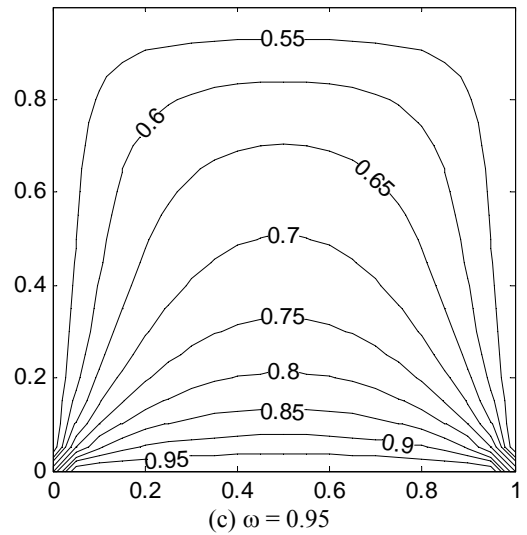
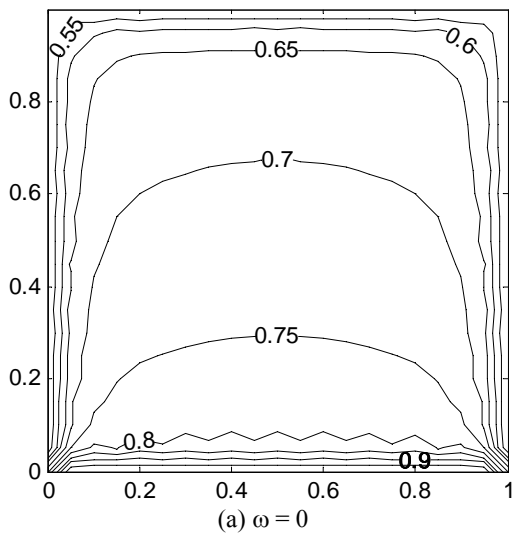
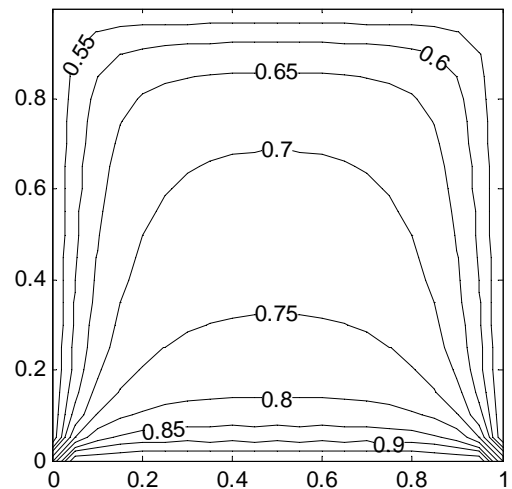


Fig 7. Isotherms for (a)  $\omega=0$ , (b)  $\omega=0.75$ , AND (c)  $\omega=0.95$  (AR=1, RC=100)



(a)  $\omega = 0$



(b)  $\omega = 0.75$

The variations of vertical centerline temperatures in a square enclosure at RC=100 for various values of single scattering albedo ( $\omega$ ) ranging from 0 to 1 are shown in Fig. 6. The vertical centerline temperatures are more at lower values of  $\omega$  and vice versa. Smaller value of  $\omega$  indicates less scattering and more absorbing capacity of the medium. So temperature of the medium increases since more energy is absorbed. The effect is more pronounced at higher values of  $\omega$  compared to that at lower values.

Exceptions occur near hot surface where crossing of temperature plots occurs (Fig. 6). Figure 7 (a) shows that the isotherms are more closely packed near all the walls for lower value of  $\omega=0$  compared to those for higher values of  $\omega$  (Figs. 7 b and c). The same is reflected in the vertical centerline temperature showing higher temperature gradient near horizontal walls (bottom and top) for lower value of  $\omega$  and vice versa. This leads to crossing of vertical centerline temperature plots. The temperature distribution is identical with that for pure conduction at  $\omega=1$ . From Eq. (7), it is clear that when  $\omega=1$ , the last term of the radiative transport equation containing temperature vanishes. So the energy conservation equation and the radiative transport equation are decoupled. As can be observed from the flow chart (Fig. 2), the upgraded solution of the radiative transport equation cannot be obtained. So the loop terminates and the temperature for the pure conduction is obtained.

### 5. CONCLUSIONS

Combined conduction and radiation heat transfer in absorbing, emitting and isotropically scattering rectangular enclosures has been studied numerically. The effect of radiation conduction parameter, aspect ratio and radiative properties on temperature distribution and heat transfer are analysed and discussed. The following

conclusions are made from the study.

- The heat loss at vertical walls through conduction affects the center temperature effectively decreasing the center temperature at higher aspect ratios.
- The temperature gradient along vertical is almost uniform when radiation conduction parameter is less, but for higher radiation conduction parameter, the gradient is comparatively more near the bottom hot wall and less near the top cold wall.
- The temperature within the enclosure is more uniformly distributed at higher values of single scattering albedo due to the scattering effect of the medium.

## 6. REFERENCES

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## 7. NOMENCLATURE

Sym bol	Meaning	Unit
AR	Aspect Ratio (H/L) (dimensionless)	(--)
G	Incident radiation	(W/m <sup>2</sup> )
G*	Incident radiation (dimensionless)	(--)
H	Height of the cavity	(m)
I	Intensity of radiation	(W/m <sup>2</sup> .sr)
k	Thermal conductivity	(W/m.K)
L	Width of the cavity	(m)
q	Heat flux	(W/m <sup>2</sup> )
Q	Heat flux (dimensionless)	(--)
RC	Radiation conduction parameter (dimensionless)	(--)
T	Temperature	K
x	Co-ordinate in x-direction	(m)
X	Co-ordinate in x-direction (dimensionless)	(--)
y	Co-ordinate in y-direction	(m)
Y	Co-ordinate in y-direction (dimensionless)	(--)
$\alpha$	Thermal diffusivity	(m <sup>2</sup> /s)
$\beta$	Volumetric co-efficient of thermal expansion	(K <sup>-1</sup> )
$\epsilon$	Emissivity of walls (dimensionless)	(--)
$\Omega$	Solid angle	(sr)
$\rho$	Density	(kg/m <sup>3</sup> )
$\sigma$	Stefan-Boltzmann constant	(W/m <sup>2</sup> .K <sup>4</sup> )
$\theta$	Temperature (dimensionless)	(--)
$\tau$	Optical thickness (dimensionless)	(--)
$\omega$	Single scattering albedo (dimensionless)	(--)

### Subscripts

b=Blackbody  
c= Conduction  
C=Cold wall  
H= Hot wall  
R=Radiation  
W=Wall